

## RECENT ADVANCES ON HYPOPLASTICITY WITH EXPLICIT ASYMPTOTIC STATES

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**Summary.** Since early days of hypoplastic model development, basic concepts of critical state soil mechanics have been included into their structure (see, by example, [1]). These models predict specific asymptotic states, in particular critical state and the isotropic asymptotic state. As a matter of fact, asymptotic states are predicted by these models for any compressive proportional strain path, but these additional states have not been controlled by the model developers and cannot be directly specified by the model users by variation of parameters (see [6]). In this talk, an approach is described allowing to incorporate any pre-defined form of asymptotic states into the hypoplastic model structure. To demonstrate the approach, a hypoplastic equivalent of the Modified Cam-clay model is presented first, originally published in [3]. Subsequently, the talk will focus on the recently developed thermo-hydro-mechanical model for partially saturated soils considering the effects of double structure. The structure of the model is explained and it is shown how the asymptotic states are explicitly incorporated into it.

### 1 EXPLICIT INCORPORATION OF ASYMPTOTIC STATES INTO HYPOPLASTICITY

The procedure for explicit incorporation of asymptotic states into hypoplasticity has first been developed in [3]. In fact, Mašín [3] inverted the procedure from Mašín and Herle [7] such that the asymptotic state boundary surface could be pre-defined and the hypoplastic model components back-calculated. The general rate formulation of the model is the same as that proposed by Gudehus [1], that is

$$\dot{\boldsymbol{\sigma}} = f_s(\mathcal{L} : \mathbf{D} + f_d \mathbf{N} \|\mathbf{D}\|) \quad (1)$$

where  $\dot{\boldsymbol{\sigma}}$  is the objective stress rate,  $\mathbf{D}$  is the Euler stretching tensor,  $\mathcal{L}$  and  $\mathbf{N}$  are fourth- and second-order hypoplastic tensors and  $f_s$  and  $f_d$  are two scalar factors.

To incorporate the asymptotic state boundary surface, Mašín [3] assumed that it changes its size with variable void ratio, but not its shape. The size of the asymptotic state boundary surface is measured by the Hvorslev equivalent pressure  $p_e$ . As in the hypoplastic model from Ref. [2], the following formulation of the isotropic normal compression line was assumed:

$$\ln(1 + e) = N - \lambda^* \ln(p/p_r) \quad (2)$$

where  $N$  and  $\lambda^*$  are parameters and  $p_r = 1$  kPa is a reference stress.  $p_e$  can thus be calculated from

$$p_e = p_r \exp \left[ \frac{N - \ln(1 + e)}{\lambda^*} \right] \quad (3)$$

Fundamental in the derivation of the hypoplastic model formulation with explicit asymptotic states is quantification of the rate of stress normalised with respect to Hvorslev equivalent pressure. An advantage is taken of the fact that at the asymptotic state the rate of the normalised stress is zero. After manipulation of hypoplastic equation, the following formulation is obtained:

$$\dot{\mathbf{T}} = f_s \mathcal{L} : \mathbf{D} - \frac{f_d}{f_d^A} \mathcal{A} : \mathbf{d} \|\mathbf{D}\| \quad (4)$$

where  $\mathcal{A}$  is a fourth-order tensor,  $f_d^A$  is a scalar factor controlling the shape of the asymptotic state boundary surface and  $\mathbf{d}$  represents the asymptotic stretching rate direction corresponding to the given stress ratio. Figure 1a shows an example of the asymptotic state boundary surface adopted in the model from [4] for various critical state friction angles.

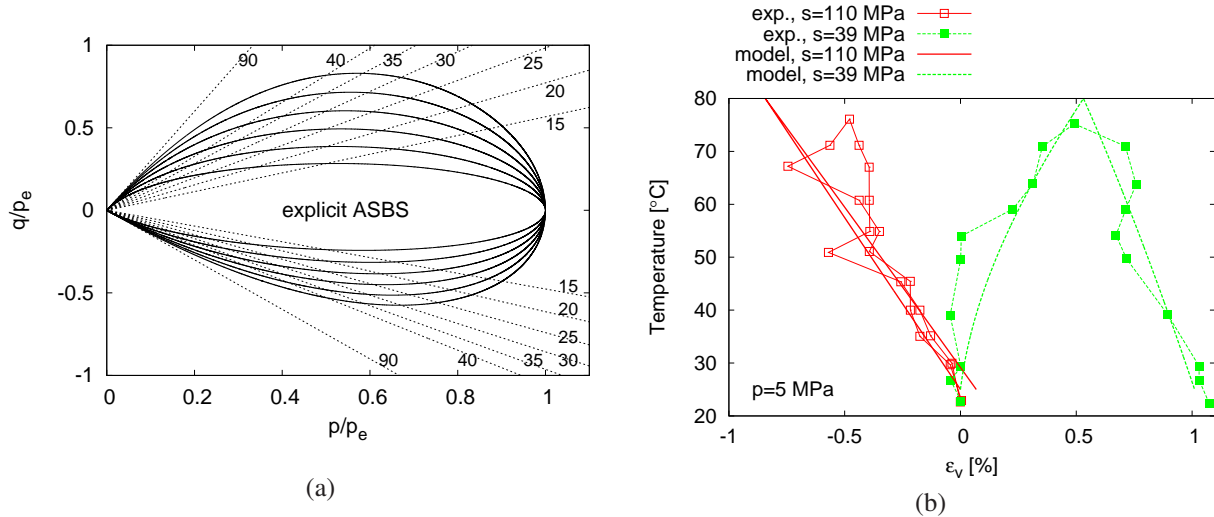


Figure 1: (a) Asymptotic state boundary surface for various critical state friction angles, (b) Predictions of the heating-cooling cycle using an advanced THM double structure model for two different suction levels.

## 2 THERMO-HYDRO-MECHANICAL MODEL CONSIDERING DOUBLE STRUCTURE

The concept of explicit state boundary surface form a basis of an advanced thermo-hydro-mechanical double structure model, which has recently been developed. The model is based on a hydro-mechanical model developed in [5]. Additionally to this model, thermal behaviour of both microstructure and macrostructure is considered, which are coupled through the double structure coupling factor. In addition, hydraulic response is coupled with the mechanical one through the dependency of water retention curve on void ratio and on temperature. Example predictions of this model (heating-cooling cycle for two different suction levels) are given in Fig. 1b.

## REFERENCES

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